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EFFECTS ARISING FROM CHARGED PARTICLE OVERCOMING OF THE LIGHT VELOCITY BARRIER

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The effects arising from accelerated and decelerated motion of the charged point particle inside the medium are studied. It is shown explicitly that in addition to the bremsstrahlung and Cherenkov shock wave, the electromagnetic shock wave arising from the charge overcoming the light velocity in the medium should be observed. This shock wave has the same singularity as the Cherenkov one and, therefore, it is more singular than the bremsstrahlung shock wave. The space-time regions where these shock waves exist and conditions under which they appear are determined.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Эффекты, возникающие при прохождении заряженными частицами скорости света в веществе

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Проанализированы эффекты, возникающие при ускоренном или замедленном движении заряженной частицы в среде. Показано, что наряду с ударной волной, связанной с началом или окончанием движения, и черенковской ударной волной существует ударная волна, возникающая в тот момент, когда скорость частицы совпадает со скоростью света в среде. Эта ударная волна обладает сингулярностью той же мощности, что и черенковская волна. Найдены пространственно-временные области и условия, при которых эта ударная волна должна наблюдаться.

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1. Introduction

Although the Vavilov–Cherenkov effect is a well established phenomenon widely used in physics and technology [1], many its aspects remain uninvestigated up to now. In particular, it is not clear how a transition from the sublight velocity regime to the superlight one occurs. Some time ago [2,3] it was suggested that alongside with the usual Cherenkov and bremsstrahlung shock waves, the shock wave associated with a charged particle overcoming the light velocity barrier should exist. The consideration presented there was pure qualitative without any formulae and numerical results. It was grounded on the analogy

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with phenomena occurring in acoustics and hydrodynamics. It seems to us that this analogy is not complete. In fact, the electromagnetic waves are pure transversal, while acoustic and hydrodynamic waves contain longitudinal components. Further, the analogy itself cannot be considered as a final proof. This fact and experimental ambiguity to distinguish the Cherenkov radiation from the bremsstrahlung one [4] make us consider effects arising from the charged particle overcoming the light velocity barrier in the framework of the completely solvable model. To be precise, we consider the straight-line motion of the point charge with a constant acceleration and evaluate the arising electromagnetic field (EMF). In accordance with Refs.2,3 we confirm the existence of the shock wave arising at the moment when charged particle overcomes the light velocity (inside the medium) barrier. This wave has essentially the same singularity as the Cherenkov shock wave. It is much stronger than the singularity of the bremsstrahlung shock wave. Previously, the accelerated motion of the point charge in a vacuum was considered by Schott [5]. Yet, his qualitative consideration was pure geometrical, not allowing the numerical investigations.

2. Statement of the Physical Problem

Let a charged particle move inside the medium with polarizabilities ε and μ along the given trajectory $\xi(t)$. Then, its electromagnetic field (EMF) at the observation point (ρ, z) is given by the Lienard–Wiechert potentials

$$\Phi(\mathbf{r}, t) = \frac{e}{\varepsilon} \sum \frac{1}{|R_i|}, \quad \mathbf{A}(\mathbf{r}, t) = \frac{e\mu}{c} \sum \frac{\mathbf{v}_i}{|R_i|}, \quad \operatorname{div} \mathbf{A} + \frac{\varepsilon\mu}{c} \dot{\Phi} = 0. \tag{2.1}$$

Here

$$\mathbf{v}_i = \left(\frac{d\xi}{dt} \right)\Bigg|_{t = t_i}, \quad R_i = |\mathbf{r} - \xi(t_i)| - \mathbf{v}_i(\mathbf{r} - \xi(t_i))/c_n$$

and c_n is the light velocity inside the medium ($c_n = c/\sqrt{\varepsilon\mu}$). Summation in (2.1) is performed over all physical roots of the equation

$$c_n(t - t') = |\mathbf{r} - \xi(t')|. \tag{2.2}$$

To preserve the causality, the time of radiation t' should be smaller than the observation time t. Obviously, t' depends on the coordinates $\mathbf{r}$, t of the point P at which the EMF is observed. With the account of (2.2) one gets for R_i

$$R_i = c_n(t - t_i) - \mathbf{v}_i(\mathbf{r} - \xi(t_i))/c_n. \tag{2.3}$$

Consider the motion of the charged point-like particle moving inside the medium with a constant accelaration along the Z axis:

$$\xi = at^2. \tag{2.4}$$

The retarded times t' satisfy the following equation

$$c_n(t - t') = [\rho^2 + (z - at'^2)^2]^{1/2}. \tag{2.5}$$

It is convenient to introduce the dimensionless variables

$$\tilde{t} = at/c_n, \quad \tilde{z} = az/c_n^2, \quad \tilde{\rho} = a\rho/c_n^2. \tag{2.6}$$

Then,

$$\tilde{t} = \tilde{t}' = [\tilde{\rho}^2 + (\tilde{z} - \tilde{t}'^2)^2]^{1/2}. \tag{2.7}$$

In order not to overload the exposition, we drop the tilda signs:

$$t - t' = [\rho^2 + (z - t'^2)^2]^{1/2}. \tag{2.8}$$

For the treated case of one-dimensional motion the denominators R_i are given by:

$$R_i = \frac{c_n^2}{a} r_i, \quad r_i = (t - t_i) - 2t_i(z - t_i^2). \tag{2.9}$$

Eq.(2.8) can be reduced to the following equation of the fourth degree

$$t'^4 + pt'^2 + qt' + R = 0. \tag{2.10}$$

Here $p = -2(z + 1/2)$, $q = 2t$, $R = r^2 - t^2$.

We consider the following two problems:

1. A charged particle rests at the origin up to a moment $t = 0$. After that it is uniformly accelerated in the positive direction of the Z axis. In this case only positive retarded times t' have a physical meaning.

II. A charged particle decelerates uniformly moving from $z = \infty$ to the origin. After the moment $t = 0$ it rests there. Only negative retarded times are physical in this case.

It is our aim to investigate space-time distribution of the EMF arising from such particle motions.

3. Particular Case

Before going to the numerical calculations it is instructive to consider a simple case corresponding to the observation point lying on the Z axis ($\rho = 0$). In this case the roots of Eq.(2.10) are given by

$$t_1 = \tau_1 - 1/2, \quad t_2 = \tau_2 + 1/2, \quad t_3 = -\tau_2 + 1/2, \quad t_4 = -\tau_1 - 1/2,$$

$$\tau_1 = \sqrt{z + t + 1/4}, \quad \tau_2 = \sqrt{z - t + 1/4}. \tag{3.1}$$

In what follows we need also the values of denominators R_i entering into the definition of electromagnetic potentials Φ, **A**:

$$r_1 = 2\tau_1(t + 1/2 - \tau_1), \quad r_2 = 2\tau_2(-t + 1/2 + \tau_2),$$

$$r_3 = 2\tau_2(t - 1/2 + \tau_2), \quad r_4 = -2\tau_1(t + 1/2 + \tau_1). \tag{3.2}$$

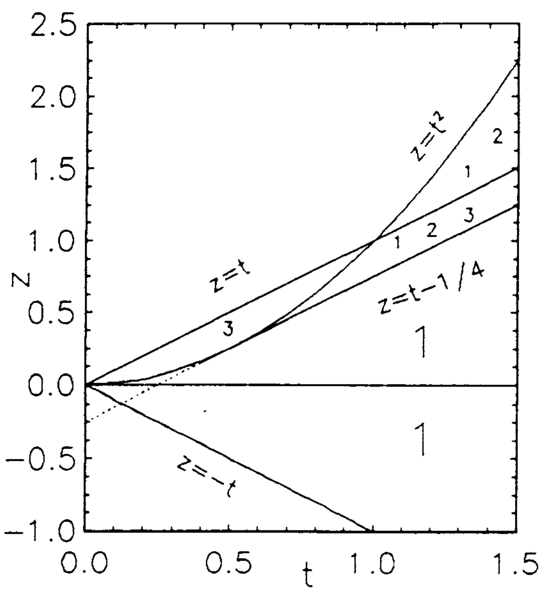


Fig.1. The space-time distribution of the retarded solutions for the particle in accelerated motion and the observation point lying on the Z axis. Numbers 1,...,3 mean the retarded solutions $t_1, \dots t_3$

Accelerated motion

For the first problem (uniform acceleration of the charged particle from the state of rest) the physical retarded times are (Fig.1):

i) t_1.

This solution exists in the space-time region $-t < z < t^2$.

ii) t_2.

This solution exists in the $t > 1/2$, $t - 1/4 < z < t^2$ region.

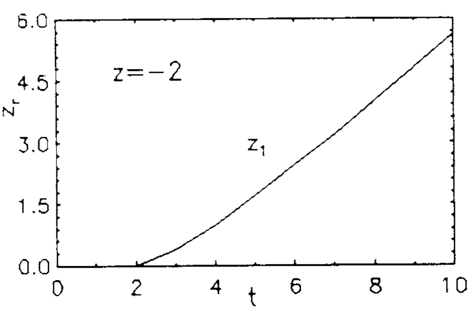
iii) t_3.

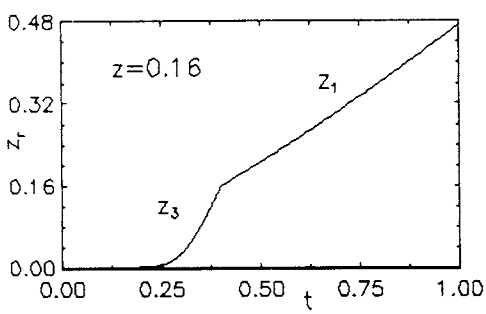
This solution exists in the regions $t < 1/2$, $t^2 < z < t$ and $t > 1/2$, $t - 1/4 < z < t$.

Let the observer be placed at a particular point of the Z axis. We clarify now what he will see at different moments of time. It is convenient to relate the current time t not to the retarded time t_r, but to the particle position z_r at that moment of time ($z_r = t_r^2$).

Consider the particular point P lying on the negative Z semi-axis (Fig.2). Up to the moment $t = -z$ the observer sees the field of the charge which rests at the origin. At the moment $t = -z$ the shock wave arising from the beginning of the particle motion arrives at P. At later times the radiation arrives from the retarded particle positions z_1 lying on the right of P.

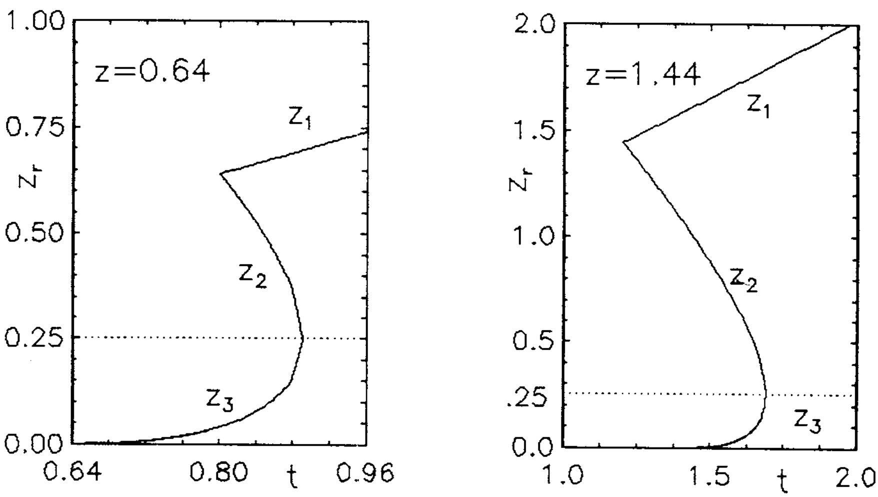
Let the observation point P lie on the positive Z semi-axis in the interval $0 < z < 1/4$ (Fig.3). Up to a moment $t = z$ the observer in P sees the electrostatic field of the charge





Figs.2,3. The retarded positions of the radiating uniformly accelerated charge as function of time for the observation point lying on the motion axis at $z = -2$ (Fig.2) and $z = 0.16$ (Fig.3) which rests at the origin. At the moment $t = z$ the bremsstrahlung shock wave from the origin reaches P. In the time interval $z < t < \sqrt{t}$ the retarded solution is t_3 which describes the radiation from the particle retarded positions lying in the interval $0 < z_3 < z$. At the moment $t = \sqrt{z}$ the charged particle reaches the observation point P. At that point R_1 and R_3 vanish and the electromagnetic potentials are infinite. For time $t > \sqrt{z}$ the observer detects the radiation from the retarded positions of the particle lying on the right of P and corresponding to t_1.

Let the observation point lie in the interval $1/4 < z < 1$ (Fig.4). Up to a moment $t = z$ the observer sees the field of the charge at rest. At the moment $t = z$ the bremsstrahlung shock wave originating from the beginning of the charge motion reaches P. In the time interval $z < t < \sqrt{z}$ the observer sees the radiation from the particle retarded positions z_3 in the interval $0 < z_r < (1 - \sqrt{z})^2$. At the moment $t = \sqrt{z}$ the charged particle (or Cherenkov shock wave) reaches the observation point. Again, electromagnetic potentials are infinite at this point. After that ($\sqrt{z} < t < z + 1/4$) the observer in P detects the radiation from three retarded positions of the particle. Two of them (z_2 and z_3) lie on the left of the observation point P and on the opposite sides of the point $z_l = 1/4$ at which the particle velocity is equal to the light velocity in the medium. As time goes, these retarded radiation positions approach z_l. At the moment $t = z + 1/4$ they fuse at the point $z_l = 1/4$ where the particle velocity equals c_n. It turns out (see (3.2)) that at this point R_2 and R_3 vanish while the electromagnetic potentials take infinite values. The disappearance of the t_2 and t_3 solutions and the infinite values of electromagnetic potentials suggest that the observation point is reached by the shock wave originating from the point $z_l = 1/4$, where the particle velocity is equal to c_n. The third of the mentioned solutions (t_1) describes the radiation from the



Figs.4,5. Same as Fig.2, but for $z = 0.64$ (Fig.4) and for $z = 1.44$ (Fig.5) particle positions lying on the right of the observation point. For $t > z + 1/4$ only this solution contributes to the observation point.

Let the observation point P lie in the region $z > 1$ (Fig.5). Up to a moment $t = \sqrt{z}$ the observer sees the electrostatic field of the charge at rest. At the moment $t = \sqrt{z}$ the charged particle (with the Mach cone accompanying it) arrives at P. The electromagnetic potentials are infinite at this moment. In the time interval $\sqrt{z} < t < z$ the observer detects the electrostatic field of the charge at rest and the radiation from two points lying on the left (z_2) and the right (z_1) of P. At the moment $t = z$ the bremsstrahlung shock wave from the origin reaches P. In the time interval $z < t < z + 1/4$ there are three retarded solutions (t_1, t_2, t_3) which contribute to P. At the moment $t = z + 1/4$ the retarded solutions t_2 and t_3 annihilate each other at the point $z_l = 1/4$ where the particle velocity is equal to c_n. This as well as infinite values of the electromagnetic potentials imply the existence of the shock wave originating from $z_l = 1/4$ point. For $t > z + 1/4$ only the radiation from t_1 solution reaches P.

Decelerated motion

In the second case (uniform deceleration of the charge up to a moment $t = 0$ after which it rests at the origin) the allowable retarded solutions are (Fig.6):

i) t_4.

This solution exists in the regions $t < -1/2$, $z > t^2$ and $t > -1/2$, $z > -t - 1/4$.

ii) t_3.

This solution exists in the regions $t < 0$, $z > t^2$ and $t > 0$, $z > t$.

iii) t_1.

This solution is defined in the regions $-1/2 < t < 0$, $-t - 1/4 < z < t^2$ and $t > 0$, $-t - 1/4 < z < -t$.

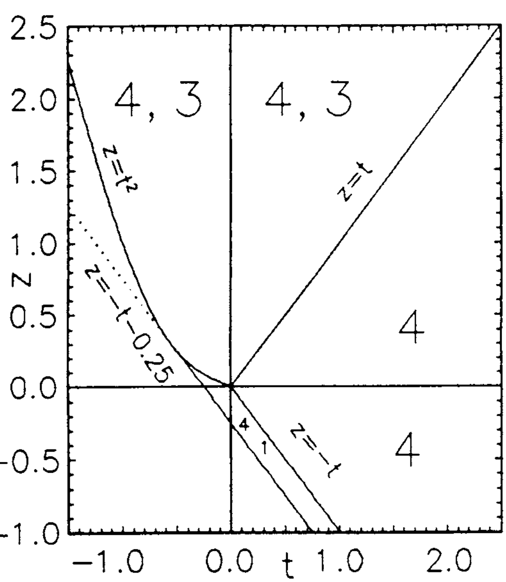
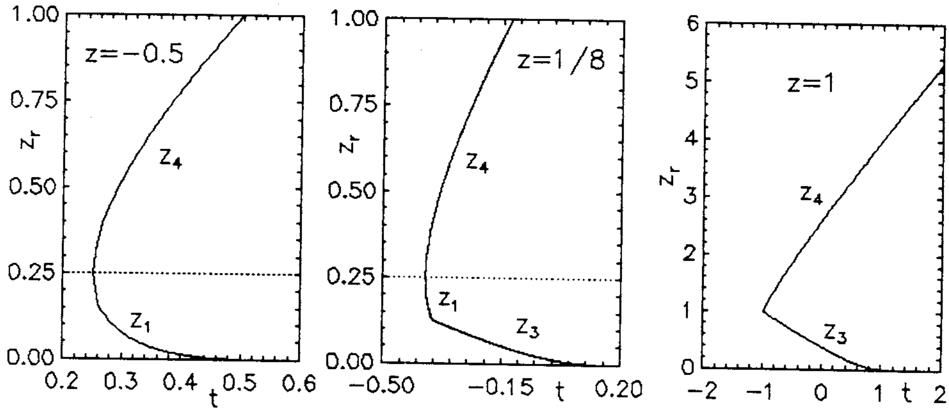


Fig.6. The space-time distribution of the retarded solutions for the particle in decelerated motion and the observation point lying on the Z axis

Let the observer be placed on the negative Z semi-axis (Fig.7). Up to a moment $t = -z - 1/4$ he does not obtain any information concerning the particle motion. At the moment $t = -z - 1/4$ the shock wave originating from the particle overcoming the light velocity barrier (at $z_l = 1/4$, $t_l = -1/2$) reaches the observation point P (the electromagnetic potentials are infinite at this point). In the time interval $-z - 1/4 < t < -z$ the observer detects the radiation from retarded charge positions (z_1 and z_4) lying on the left and right of z_l. At the moment $t = -z$ the observer detects the shock wave arising from the termination of the particle motion. For $t > -z$ the observer



Figs.7–9. The retarded positions of the radiating uniformly decelerated charge as functions of time for the observation points lying on the motion axis at $z = -0.5$ (Fig.7), $z = 1/8$ (Fig.8), and $z = 1$ (Fig.9)

sees the electrostatic field of the charge which rests at the origin and the radiation from the remote retarded positions of the charge.

Let the observation point lie within the interval $0 < z < 1/4$ (Fig.8). At the moment $t = -z - 1/4$ the shock wave originating from the particle overcoming the light velocity barrier (at $z = z_l$) reaches the observer. Again, the electromagnetic potentials are infinite at this moment. In the time interval $-z - 1/4 < t < -\sqrt{z}$ the radiations from two retarded positions of the charge (z_4 and z_1) arrive at P. They lie on different sides of z_l, on the right of the observation point z. As time goes, one of the radiating points (z_1) approaches the origin, while the other (z_4) moves away from z. At the moment $t = -\sqrt{z}$ the electromagnetic potentials become infinite as the particle arrives at P. At this moment the t_1 solution disappears, but, instead, t_3 arises. In the time interval $-\sqrt{z} < t < z$ the observer sees the radiation from two points lying on different sides of him. At the moment $t = z$ one of the retarded positions of the charge (z_3) comes to the origin and the corresponding bremsstrahlung shock wave reaches the observer. For times $t > z$ the observer sees the electrostatic field of the charge at rest and the radiation field from the remote retarded positions z_4 of the charge.

Let the observer be placed at the point P with $z > 1/4$ (Fig.9). There is no field in P up to a moment $t = \sqrt{z}$. At this moment the charge with accompanying it Cherenkov shock wave arrives at P. After that the observer sees the radiation field from two retarded positions lying on different sides of P. As time goes, one of the retarded positions (z_3) approaches the origin, while the other (z_4) goes away. At the moment $t = z$ the observer sees that charge reaches the origin and detects the shock wave associated with the termination of the particle motion. After that moment the observer detects the electrostatic field of the charge which rests at the origin and the radiation field from one remote retarded position z_4 of the charge.

Concluding this section we note the existence of two types of the shock waves. The bremsstrahlung shock waves associated with the beginning or termination of the charge motion correspond to the finite jumps of electromagnetic potentials. Therefore, the field strengths have the δ-type singularities. On the other hand, the Cherenkov shock wave and the shock wave associated with the charged particle overcoming the light velocity barrier correspond to infinite jumps of electromagnetic potentials (due to the vanishing of the denominators R_i). Thus, they carry a much stronger singularity.

4. Numerical Results

Numerical results were obtained by solving [6] Eq.(2.10) under the condition $t' < t$. We consider at first the typical case corresponding to $|t| = 2$.

Accelerated motion

For the first of the treated problems (uniform acceleration of the charge which initially rests at the origin) the resulting configuration of the shock waves is shown in Fig.10. We see on it the Cherenkov shock wave $C_M^{(1)}$, the shock wave $C_L^{(1)}$ closing the Mach cone and the sphere C_0 representing the spherical shock wave arising from the beginning of the charge motion. It turns out that the surface $C_L^{(1)}$ with a high accuracy is approximated by the part of the sphere $\rho^2 + (z - 1/4)^2 = (t - 1/2)^2$ (shown by the short-dash curve C) which corresponds to the shock wave emitted from the point $z = 1/4$ at the moment $t = 1/2$ when the velocity of the charged particle coincides with the velocity of light in the medium. On the internal sides of the surfaces $C_L^{(1)}$ and $C_M^{(1)}$ electromagnetic potentials acquire infinite values. On the external side of $C_M^{(1)}$ lying outside C_0 the electromagnetic potentials are zero (as there are no solutions there). On the external sides of $C_L^{(1)}$ and on the part of the $C_M^{(1)}$ surface lying inside C_0 the electromagnetic potentials have finite values.

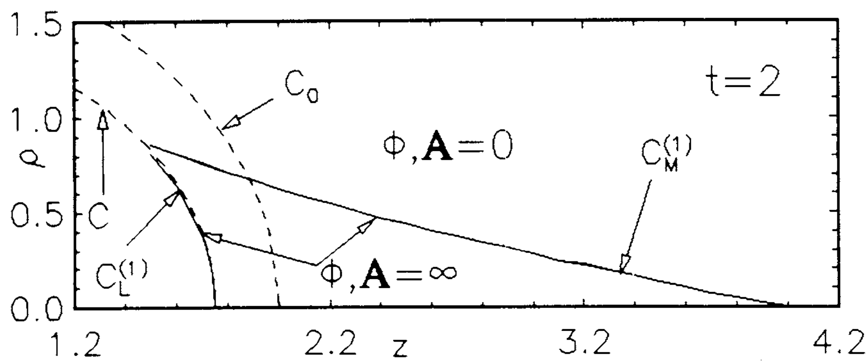


Fig.10. The distribution of the shock waves for the uniformly accelerated charge and $t = 2$. The short-dash curve C represents the spherical shock wave emitted from the point $z = 1/4$ at the moment $t = 1/2$

Decelerated motion

Now we turn to the second problem (uniform deceleration of the charged particle along the positive z semi-axis up to a moment $t = 0$ after which it rests at the origin). In this case only negative retarded times t_i have a physical meaning.

For the observation time $t > 0$ the resulting configuration of the shock waves is shown in Fig.11. We see the bremsstrahlung shock wave C_0 arising from the termination of the charge motion and the blunt shock wave $C_L^{(2)}$. Its head part with a high accuracy is described by the sphere $\rho^2 + (z - 1/4)^2 = (t + 1/2)^2$ (shown by the short-dash curve) corresponding to the shock wave emitted from the point $z = 1/4$ at the moment $t = -1/2$ when the velocity of the decelerated charged particle coincides with the velocity of light in the medium. The electromagnetic potentials vanish outside $C_L^{(2)}$ (as no solutions exist there) and acquire infinite values on the internal part of $C_L^{(2)}$ (due to vanishing of the denominators R_1 and R_2 there). Therefore, the surface $C_L^{(2)}$ represents the shock wave. As a result, for $t > 0$, $t' < 0$ one has the shock wave $C_L^{(2)}$ and the bremsstrahlung shock wave C_0 arising from the termination of the particle motion.

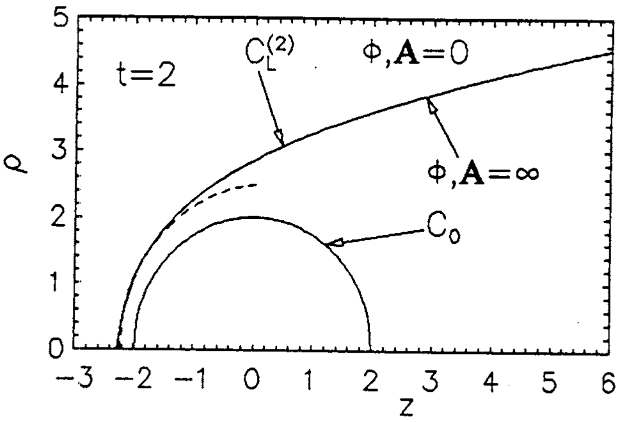


Fig.11. The distribution of the shock waves for the uniformly decelerated charge and $t = 2$. The short-dash curve represents the spherical shock wave emitted from the point $z = 1/4$ at the moment $t = -1/2$

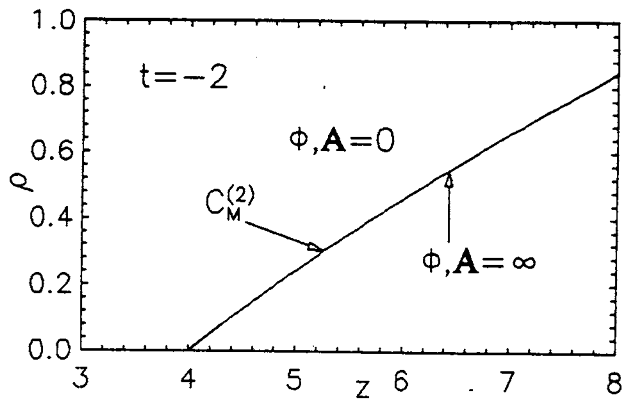


Fig.12. The distribution of the shock waves for the uniformly decelerated charge and $t = -2$

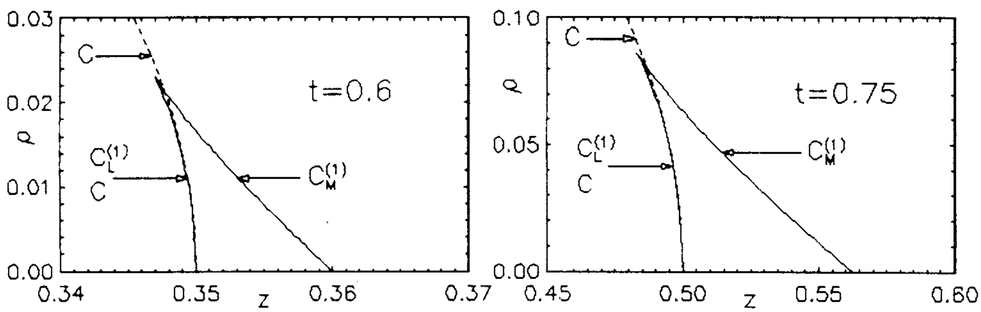
For $t < 0$, $t' < 0$ the physical solutions exist only inside the Mach cone $C_M^{(2)}$ (Fig.12). On its internal boundary the electromagnetic potentials acquire infinite values. On the external boundary the electromagnetic potentials are zero (as no solutions exist there). Thus, for the case of decelerated motion and the observation time $t = -2$ the physical solutions are contained inside the Mach cone $C_M^{(2)}$ (Fig.12).

5. Discussion

Consider at first the accelerated motion of the charge beginning from the origin at the moment $t = 0$. The time evolution of the arising shock waves is shown in Figs.13–15. All the Mach cones shown in Figs.13–15 exist only for $t > 1/2$, $z > 1/4$. This means that the observer being placed in the space region with $z < 1/4$ will not see either the Cherenkov shock wave or the shock wave originating from the overcoming the light velocity barrier in any moment of time. Only the shock wave C_0 (not shown in Figs.13–16) associated with the beginning of the charge motion reaches him at the moment $c_n t = r$. Moreover, the detection of the aforementioned shock waves ($C_L^{(1)}$ and $C_M^{(1)}$) in the $z > 1/4$ region is possible if the distance ρ from the Z axis satisfies the equation

$$\rho < \rho_c, \quad \rho_c = \frac{4}{3\sqrt{3}}\left(z - \frac{1}{4}\right)^{3/2}, \quad z > \frac{1}{4}. \tag{5.1}$$

Inside this region the observer sees at first the Cherenkov shock wave $C_M^{(1)}$. Later he detects the bremsstrahlung shock wave C_0 and the shock wave $C_L^{(1)}$ associated with overcoming the light velocity barrier. It is remarkable that the surface of the $C_L^{(1)}$ shock wave with a high accuracy coincides with the surface of the sphere $\rho^2 + (z - 1/4)^2 = c_n^2(t - 1/2)^2$ describing the spherical wave emitted by the charge from the point $z = 1/4$ at the moment $t = 1/2$ when the charge velocity is equal to c_n. These spheres are shown by the short-dash curves in Figs.13–15. Outside the region defined by (5.1) the observer sees only the bremsstrahlung shock wave C_0 which reaches him at the moment $c_n t = r$. Further, for $t < 1/2$ only one retarded solution (t_1) exists. It is confined to the sphere C_0 of the radius $r = c_n t$.



Figs.13,14. The positions of the Cherenkov shock wave $C_M^{(1)}$ and the shock wave $C_L^{(1)}$ arising from the charge overcoming of the light velocity barrier for the accelerated charge are shown for the moment $t = 0.6$ (Fig.13) and $t = 0.75$ (Fig.14). Short dash curve C represents the spherical wave emitted from the point $z = 1/4$ at the moment $t = 1/2$ when the accelerated charged particle overcomes the light velocity barrier

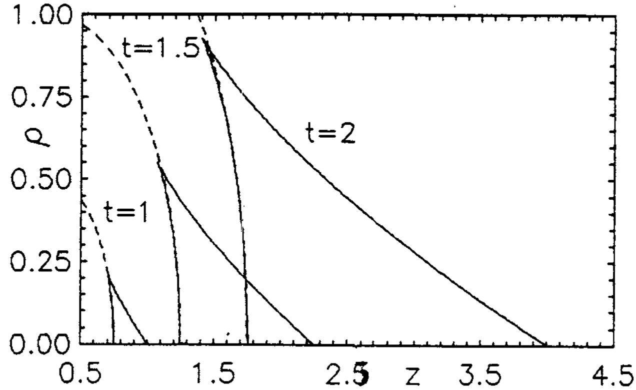


Fig.15. The same as in Fig.13, but for $t = 1$, 1.5 and $t = 2$

Therefore, the observer will not detect either the Cherenkov shock wave or that of originating from the overcoming light velocity barrier. The dimensions of the Mach cones shown in Figs.13–15 are zero for $t = 1/2$ and continuously rise with time for $t > 1/2$. The physical reason for this behaviour is that $C_L^{(1)}$ shock wave closing the Mach cone propagates with the light velocity c_n, while the head part of the Mach cone (i.e., the Cherenkov shock wave $C_M^{(1)}$) attached to the charged particle expands with the velocity $v > c_n$.

In the gas dynamics the existence of at least two shock waves attached to the finite body moving with a supersonic velocity was proved on the very general grounds by Landau and Lifshitz ([10], Chapter 13). In the present context we associate them with $C_L^{(1)}$ and $C_M^{(1)}$ shock waves.

For the decelerated motion (see Fig.16) the observer in the space region $z < 0$ detects the blunt shock wave $C_L^{(2)}$ first and the bremsstrahlung shock wave C_0 later. It turns out that the head part of this blunt wave with a high accuracy coincides with the sphere $\rho^2 + (z - 1/4)^2 = (t + 1/2)^2$ describing the spherical wave emitted from the point $z = 1/4$ at the moment $t = -1/2$ when the charge velocity coincides with c_n. The observer in the $z > 1/2$ region detects only the Cherenkov shock wave $C_M^{(2)}$.

In order not to hamper the exposition, we did not mention, in this section, the continuous radiation which reaches the observer between the arrival of two shock waves or after the arrival of the last shock wave. It is easily restored either from the simplified case considered in Sec.3 or from Figs.10–12.

However, some precaution is needed. For the motion law (2.4) the charge velocity may exceed c, the velocity of light in vacuum. Consider first the accelerated motion. The external 4-force maintaining the accelerated motion (2.4) becomes infinite (due to the γ-factor ($\gamma = (1 - \beta^2)^{-1/2}$) in it). Therefore, this motion cannot be realized for v close to c. In any case, the effects arising from the proximity of charge velocity to c do not produce any

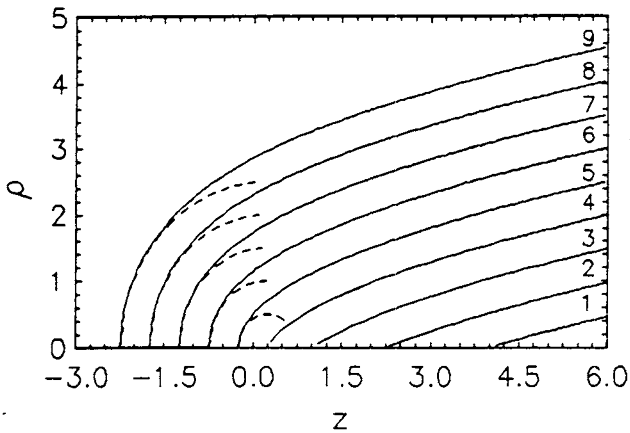


Fig.16. The continuous transformation of the Cherenkov shock wave $C_M^{(2)}$ (1) into the blunt shock wave $C_L^{(2)}$ (9) for the decelerated motion. The numbers 1–9 refer to the moments of time $t = -2; -1.5; -1; -0.5; 0; 0.5; 1; 1.5$, and 2, resp. Short-dash curves represent the spherical waves emitted from the point $z = 1/4$ at the moment $t = -1/2$ when the decelerated charged particle overcomes the light velocity barrier

discontinuties and will be observed after the arrival of the last of the shock waves considered earlier. The situation is slightly more complicated for the decelerated motion. To escape the troubles with $v > c$ one may imagine that the charged particle is at rest at the point $z = -z_0$ up to a moment $t = -t_0$, after which it instantly acquires the velocity $c_n < v < c$. After the moment $t = -t_0$ the charge moves towards the origin according to a law similar to (2.4). The radiation field arising from such a velocity jump was studied in [9]. It turns out that the arising physical picture insignificantly differs from that considered in previous sections. Let the observation point P lie in the negative Z semi-space. Then, after the arrival of the $C_L^{(2)}$ shock wave, the shock wave C_1 associated with the beginning of the charge motion (at $t = -t_0$) arrives at P. For the observation point P in the positive Z semi-space (more accurately, for $z > 1/4$) the shock wave C_1 reaches P after the arrival of the Cherenkov shock wave $C_M^{(2)}$. In both cases the C_1 shock wave closes either the blunt shock wave $C_L^{(2)}$ or the Mach cone $C_M^{(2)}$ (likewise the shock wave $C_L^{(1)}$ shown in Figs.10–14 closes the Mach cone $C_M^{(1)}$). The singularity of the C_1 shock wave is the same as the singularity of C_0 shock wave and, therefore, is weaker than the singularity either of C_M or C_L.

So far we have considered the physical effects arising when the velocity of the point-like charged particle continuously passes through the light-velocity barrier. The electromagnetic fields of the uniformly moving charge are well-known both for $v > c_n$ and

$v < c_n$ [5,7,8]. But what happens if the particle velocity exactly coincides with the light velocity in the medium c_n? (This question was posed by Prof. Tyapkin). For this case the equation defining t' is

$$c_n(t - t') = [\rho^2 + (z - c_n t')^2]^{1/2}.$$

Solving it relative to t' one gets

$$c_n t' = \frac{1}{2}\frac{r^2 - c_n^2 t^2}{z - c_n t}.$$

The nonvanishing components of the electromagnetic potentials are equal to

$$\Phi = \frac{e\Theta(c_n t - z)}{\varepsilon(c_n t - z)}, \quad A_z = \frac{ec_n \mu\Theta(c_n t - z)}{c(c_n t - z)}.$$

As $\mathbf{A}$ and Φ do not depend on the cylindrical coordinates ρ and ϕ, so $\mathbf{B} = \mathbf{H} = E_\rho = E_\phi = 0$ and

$$E_z = -\frac{\partial\Phi}{\partial z} - \frac{1}{c}\frac{\partial A_z}{\partial t},$$

$$\frac{\partial\Phi}{\partial z} = -\frac{e\delta(c_n t - z)}{\varepsilon(c_n t - z)} + \frac{e\Theta(c_n t - z)}{\varepsilon(c_n t - z)^2}, \quad \frac{1}{c}\frac{\partial A_z}{\partial t} = \frac{e\delta(c_n t - z)}{\varepsilon(c_n t - z)} - \frac{e\Theta(c_n t - z)}{\varepsilon(c_n t - z)^2}.$$

It turns out that $\mathbf{E}$ and $\mathbf{H}$ vanish everywhere except, possibly, the plane $z = c_n t$. In it, E_z reduces to the difference of two infinities and the final answer remains to be undetermined. However, the integral of $\mathbf{E}$ taken over an arbitrary closed surface surrounding the charge should be equal to $4\pi e$. As $\mathbf{E}$ is entirely confined to the plane $z = c_n t$, it should be infinte on this plane (to guarantee the finiteness of the above integral). As a result, the electromagnetic field of the particle moving with the velocity coinciding with the light velocity in the medium differs from zero only on the plane normal to the axis of motion and passing through the charge itself. The same ambiguity arises if one takes the formulae describing the charge motion with $v > c_n$ (see, e.g., [9]) and will tend $v \to c_n$ in them. We observe that for $v = c_n$ the shock wave coincides with the $z = c_n t$ plane, i.e., it has an infinite extension. The same effect takes place in gas dynamics when the velocity of the body coincides with the velocity of sound ([10], Chapter 12).

6. Conclusion

To the end, we confirm the qualitative predictions of Refs. 2,3 concerning the existence of the shock waves arising from the charge overcoming the light velocity barrier (inside the medium). It would be interesting to observe them experimentally.

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